Introduction to quantum information

WS 2012/13 **19.11.2012**
Assignment 3 **Due date 28.11.2012**

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Problem 1 *Universal gates*

a) Consider a qubit initially in the state $|\psi\rangle = a|0\rangle + b|1\rangle$. Calculate the state after (i) the rotation $R_x(\theta) = e^{-i\theta\sigma_x/2}$, the rotation (ii) $R_z(\theta) = e^{-i\theta\sigma_z/2}$, and (iii) the Hadamard gate H. Give an intuitive explanation on what these gates do. *(1 point)*

b) Show explicitly that the SWAP gate between two qubits does not create entanglement. *(1 point)*

c) Give a matrix representation of the square root of SWAP. Is it unique (excluding the trivial non-uniqueness of the global phase)? Show that this gate, together with single-qubit unitary gates is universal. You can use the fact that CNOT together with single-qubit unitary gates is universal.
Hint: One possible procedure to obtain CNOT uses gates in the following sequence: H-$R_z(\pi/2)$-$R_z(-\pi/2)$-$\sqrt{\text{SWAP}}$-$R_z(\pi)$-$\sqrt{\text{SWAP}}$-H *(with the gates on the right applied first). Show how to apply the gates on the two qubits.* *(3 points)*

d) Give an example of a factorized two-qubit state that is turned into an entangled state by iSWAP. Show that iSWAP, together with single-qubit unitary gates, is universal. *(3 points)*

Problem 2 *Hamiltonians*

a) Two qubits interact by the Ising interaction, $\hat{H}_I = J(t)\hat{Z}_1\hat{Z}_2$. What is the most natural two-qubit gate to use for a universal set? *(1 point)*

b) Same as above for $\hat{H}_I = J(\hat{X}_1\hat{X}_2 + \hat{Y}_1\hat{Y}_2)$. It is recommended to first find the matrix elements of this Hamiltonian explicitly before exponentiating it. *(2 points)*

Problem 3 *Deferred measurement*

Consider two qubits and compare the following two circuits: i) We apply a controlled unitary $\hat{U}$ with the first qubit being the control, then we measure the first qubit in the computational

($\hat{Z}$) basis and ii) We measure the first qubit in the $\hat{Z}$ basis and then apply $\hat{U}$ to the second qubit if and only if the measurement yields 1. Compute the Kraus map for both procedures. Under what condition are they the same? *(2 points)*

Problem 4 *Teleportation*

a) A horizon in general relativity is defined as a surface in spacetime that cannot be crossed by anything, not even light. It surrounds, e.g., a black hole. Is it possible to teleport quantum states across a horizon? *(1 point)*

b) Show that the circuit below performs quantum teleportation without measurement. *(2 points)*