Introduction to quantum information

WS 2012/13 **15.1.2013**
Assignment 6 **Due date 25.1.2013**

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Problem 1 *Eigenvalue estimation*

a) The quantum Fourier transformation has applications other than order finding. This problem leads you through the steps for one of them.

Let U be an N-qubit unitary operator. As such, it has 2^N eigenvalues of unit size $z_n = e^{i\varphi_n}$ with corresponding eigenstates $|\psi_n\rangle$ such that $U|\psi_n\rangle = z_n|\psi_n\rangle = e^{i\varphi_n}|\psi_n\rangle$, and $U^{2^j}|\psi_n\rangle = e^{i\varphi_n 2^j}|\psi_n\rangle$.

z_n can be estimated on an $N+k$ qubit register by:
(i) first preparing the k-qubits in their ground state and preparing the N-qubit state $|\psi_n\rangle$,
(ii) creating an equal superposition amongst the k qubits,
(iii) performing k controlled$-U^{2^{j-1}}$ gates for $j = 1, 2, .., k$ where the jth qubit in the k-qubit register is the control and the N-qubit register is the target,
(iv) performing an inverse quantum fourier transform on the k-qubit register, and
(v) measuring the k-qubit register.
Draw a circuit diagram representing this algorithm. *(1 point)*

b) Show that the probability of measuring the k-qubit register in state y is

$$P(y) = \frac{1}{2^{2k}} \left| \frac{e^{i2^k\varphi_n} - 1}{e^{i(\varphi_n - 2\pi y/2^k)} - 1} \right|^2. \tag{1}$$

For full credit, the state after each of steps (i)-(v) should be given. *(2 points)*

c) Plot $P(y)$ for all allowed values of y in the case $\varphi_n = \pi/2$, for $k = 1, 2, 3, 4$. You should have a distribution of 2^k probabilities for each k. Plot the success probability as a function of k in this case. *(1 point)*

d) Repeat c) in the case that $\varphi_n = 3$, for $k = 1, 2, 3, 4$. Why does this case converge differently than in c)? *(1 point)*

e) How does the precision of your estimate depend on the number of controlled-U operations used in the circuit? Note that one application of controlled-$U^{2^{j-1}}$ counts as 2^{j-1} applications of a controlled-U. *(1 point)*

Problem 2 *Extra credit: Implementations of Shor's algorithm*

Shor's factoring algorithm has been implemented experimentally to factor 15 using nuclear magnetic resonance [Vandersypen *et al.*, Nature 414: 883887 (2001), arXiv:quant-ph/0112176] and superconducting quantum processors [Lucero *et al.*, Nature Physics 8, 719-7235 (2012); arXiv:1202.5707] (you can download arXiv papers from everywhere and the actual publications while you are connected on campus). Study the algorithm that has been implemented. Is it fully generic, or have the authors cheated? *(4 points)*

Problem 3 *Grover algorithm*

a) Consider the two-qubit Grover algorithm. Design a quantum circuit for an oracle that tags the third entry (binary 10). *(1 point)*

b) Assume that in the two-qubit Grover algorithm, both states 0 and 3 (00 and 11) are tagged. Design a quantum circuit for this oracle. What will you measure? *(1 point)*

c) *Extra Credit:* Plot the probability of success as a function of number of repetitions of Grover's algorithm for both cases described above. Compare the scaling. *(2 points)*