Introduction to quantum information

WS 2012/13 **22.1.2013**
Assignment 7 **Due date 6.2.2013**

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Problem 1 *Quantum error models*

The Bloch equation in a simplified case leads to the following time evolution for a two-state density matrix $\rho = \frac{1}{2}(\hat{1} + \sum_i m_i \hat{\sigma}_i)$

$$\dot{m}_z = -m_z/T_1, \quad (1)$$
$$\dot{m}_{x/y} = -m_{x/y}/T_2. \quad (2)$$

This equation describes decoherence of a single-qubit quantum memory.

a) Compute the formal solution for arbitrary initial $m_i(0)$. *(1 point)*

b) Rewrite the solution after time $t > 0$ using Kraus operators and verify that this is a combination of phase- and bit-flip errors. *(3 points)*

Problem 2 *Error correcting codes*

a) The condition that different errors $\hat{E}_a$ lead to distinguishable syndromes on the logical qubits $|i\rangle_L$ can be written as ${}_L\left\langle i \left| \hat{E}_a^\dagger \hat{E}_b \right| j \right\rangle_L = C_{ab}\delta_{ij}$ where the coefficients C_{ab} form an arbitrary matrix. Show that the single-qubit repetition code satisifies this condition for single bit-flip errors. *(1 point)*

b) The seven-qubit Steane code is a clean and efficient code to correct single-qubit phase and bit-flip errors. The codewords are $|0\rangle_L = (|0000000\rangle + |1010101\rangle + |0110011\rangle + |1100110\rangle + |0001111\rangle + |1011010\rangle + |0111100\rangle + |1101001\rangle)/\sqrt{8}$ and $|1\rangle_L = \hat{X}^{\otimes 7}|0\rangle_L$. Show that these are eigenstates of $\hat{S}_1 = \hat{1}\hat{1}\hat{1}\hat{X}\hat{X}\hat{X}\hat{X}$ and $\hat{S}_5 = \hat{1}\hat{Z}\hat{Z}\hat{1}\hat{1}\hat{Z}\hat{Z}$. *(1 point)*

c) Below (next page) is the complete list of the syndrome operators S_i. If an arbitrary superposition of $|0\rangle_L$ and $|1\rangle_L$ is affected by i) bit flip on qubit 3 and ii) phase flip on qubit 6, show that the resulting state is still an eigenstate of the syndrome operators. What are the eigenvalues? *(2 points)*

Element	Operator
S_1	$IIIXXXX$
S_2	$IXXIIXX$
S_3	$XIXIXIX$
S_4	$IIIZZZZ$
S_5	$IZZIIZZ$
S_6	$ZIZIZIZ$