Introduction to quantum information processing
Exercise sheet 3

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***Note**: You may hand in your solutions in a group with up to three persons. Please provide your name to your solutions.*

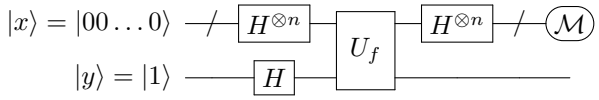
Exercise 1: Another quantum algorithm (*12 points*)

Suppose that we are given a black box which takes n (qu)bits $|x_1 x_2 \dots x_n\rangle$ as input and evaluates the function $f : \{0,1\}^n \to \{0,1\}; x \mapsto x \cdot s \pmod 2$ with an n digit string $s \in \{0,1\}^n$, *but we do not know* s. Our task is now to determine the secret string s.

(a) Describe shortly a procedure of how to determine s with a classical computer. How often does the function f_s need to be evaluated? (*3 points*)

Let us try to tackle that problem with a quantum computer. Before doing so, we first need to implement a unitary circuit containing a unitary variant of the black box. Suppose that the person knowing the secret string s already realized the black box and supplies the corresponding unitary gate U_f to us with its action on a state $|x\rangle\,|y\rangle$ being $U_f\,|x\rangle\,|y\rangle = |x\rangle\,|y \oplus f(x)\rangle$.

(b) Consider the given quantum circuit and give the current state $|x\rangle\,|y\rangle$ of the n register qubits $|x\rangle$ and the ancilla qubit $|y\rangle$ after each step from left to right. Explain shortly, how one can deduce the unkown string s by a single run of the algorithm.



(*9 points*)

Exercise 2: Quantum Fourier transformation (*8 points*)

Give the quantum circuit for the QFT and its matrix representation for $n = 3$ qubits explicitly.