

Introduction to quantum information

WS 2012/13
Assignment I

17.10.2012
Due date 24.10.2012

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<http://qsolid.uni-saarland.de/?Lehre>

Problem 1 *Trace*

- a) Prove that the trace has cyclic property $\text{Tr}(\hat{A}\hat{B}\hat{C}) = \text{Tr}(\hat{B}\hat{C}\hat{A})$. (1 point)
- b) Show that the trace is basis-independent. (2 points)

Problem 2 *Tensor products*

- a) Compute a Schmidt decomposition of $|\psi\rangle = \frac{1+\sqrt{6}}{2\sqrt{6}}|00\rangle + \frac{1-\sqrt{6}}{2\sqrt{6}}|01\rangle + \frac{\sqrt{2}-\sqrt{3}}{2\sqrt{6}}|10\rangle + \frac{\sqrt{2}+\sqrt{3}}{2\sqrt{6}}|11\rangle$. (2 points)
- b) Prove that $(|00\rangle + |11\rangle)/\sqrt{2}$ is entangled (verschränkt). (1 point)

Problem 3 *Logic operation*

- a) Show that the CNOT can be interpreted as a binary adder modulo 2. (1 point)

Initial		After CNOT	
Control	Target	Control	Target
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

- b) Describe a reversible 4-bit circuit that adds the integer $y \in \{0, 1, 2, 3\}$ represented in binary in the first bits to the integer z represented in binary in the last two bits. (2 points)

Problem 4 *Universal gates*

- a) Construct OR, XOR, and AND from NAND and FANOUT. *(5 points)*
- b) The Fredkin gate is a three-bit gate that swaps bits 2 and 3 if and only if bit 1 is in state 1. Show that it is universal. *(2 points)*